Interference of Bose-Einstein condensates and entangled single-atom state in a spin-dependent optical lattice

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Abstract. We present a theoretical model to investigate the interference of an array of Bose-Einstein condensates loaded in a one-dimensional spin-dependent optical lattice, which is based on an assumption that for the atoms in the entangled single-atom state between the internal and the external degrees of freedom each atom interferes only with itself. Our theoretical results agree well with the interference patterns observed in a recent experiment by Mandel et al. [Phys. Rev. Lett. **91**, 010407 (2003)]. In addition, an experimental suggestion of nonuniform phase distribution is proposed to test further our theoretical model and prediction. The present work shows that the entanglement of a single atom is sufficient for the interference of the condensates confined in a spin-dependent optical lattice and this interference is irrelevant with the phases of individual condensates, i.e., this interference arises only between each condensate and itself and there is no interference effect between two arbitrary different condensates.

1 Introduction

Since the first interference measurement [1] on two expanding condensates there has been growing interest in the experimental and theoretical study of the interference of Bose-Einstein condensates (BECs). In particular, optical lattices created by retroreflected laser beams provide a unique tool for testing at a fundamental level the quantum properties of BECs in a periodic potential [2]. The interference patterns obtained from the expansion of an array of condensates trapped in an optical lattice are commonly used as a probe of the phase properties of this system [3–6]. Current understanding of the interference of BECs is largely based on the concept of phase coherence which reveals the superfluidity and the matter wave nature of the condensates.

For the interference of two condensates, it is shown that an interference pattern arises whether they are initially in phase state (with locked relative phase) [1] or in Fock state (with definite particle number) [7–10]. In the latter case, the interference effect still originates from a

well-defined relative phase which is "built up" during the sequence of measurements, i.e., the definite phase is derived from the dynamic evolution of this system (initially with random relative phase). For a fully coherent array of condensates in optical lattices, the interference pattern obtained from the free expansion is a natural result of the fixed relative phases between different condensates belonging to consecutive wells [3–5,11–16]. When the coherent array of condensates enter the Mott insulating phase (MIP) [17,18] in which phase coherence is lost, things become complicated. The pioneering experiment [17] of the superfluid to Mott-insulator transition demonstrated that when the weakly interacting gas entered the MIP the interference pattern became blurry and even disappeared completely. Whereas for a strongly interacting gas it has been pointed out that a good measure for this Mott transition was excitation spectra rather than interference patterns [19]. Besides, relevant theoretical works [20,21] based on a correlation function method imply that even in the MIP, an interference pattern should also be observed in a single measurement. On the other hand, a recent interference experiment by Hadzibabic et al. [22] states that the periodicity of the optical lattice is sufficient for the interference of an array of independent BECs even with no phase coherence. A similar discussion is also found in

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a theoretical reference [23]. It is therefore important to explore the physics of interference patterns produced following the release and expansion of BECs.

In this paper, we present a theoretical model to investigate the interference of an array of BECs confined in a spin-dependent optical lattice, which is motivated by a recent experiment by Mandel et al. [24]. The interference patterns obtained from our theoretical model and numerical calculations agree well with those observed in the experiment [24]. Our conclusion is that the interference of an array of Bose condensates trapped in a spin-dependent optical lattice results from the entanglement of a single atom and the interference is irrelevant with the phases of individual condensates.

This paper is organized as follows: in Section 2, after introducing the basic spirit of the experiment [24] a theoretical model is presented. In Section 3, we calculate numerically the density distributions of the wave packets in spin states $|1\rangle$ and $|0\rangle$, respectively. Then we compare the theoretical interference patterns with those observed in the experiment. Section 4 deals with an experimental suggestion of nonuniform phase distribution, which can be used to test further the theoretical model and prediction. Finally, discussion and conclusion is given in Section 5.

2 Theoretical model

For a Bose-condensed gas in a harmonic magnetic trap and a three-dimensional (3D) optical lattice, the atoms are localized on individual lattice sites when the system enters the MIP. After switching off the magnetic trap and the lattice potentials along the y- and z-directions there only exists a 1D spin-dependent optical lattice along the x-direction which is formed by two counterpropagating laser beams with linear polarization vectors enclosing an angle θ. Then each atom is prepared into a coherent superposition of two spin states $|0\rangle \equiv |F = 1, m_F = -1\rangle$ and $|1\rangle \equiv |F = 2, m_F = -2\rangle$ using a microwave pulse. By changing the polarization angle θ one can realize the splitting and transport of atomic wave packets such that the wave packets of an atom in spin states $|0\rangle$ and $|1\rangle$ respectively are transported in opposite directions, which is the so-called spin-dependent transport. Finally, the optical lattice is turned off and the emerging interference patterns can be used as a diagnosis signal for the coherence of the spin-dependent transport. In a word, the spin-dependent transport is the principal idea of the experiment by Mandel et al. [24].

Our starting point is that there is an array of Bose condensates formed in the 1D spin-dependent optical lattice along the horizontal x-direction when the magnetic trap and the lattice potentials along the y- and z-directions are turned off. Since the system experienced a Mott transition in advance these condensates do not have any phase coherence relative to each other any more. In this situation, the tunnelling between neighboring lattice sites is suppressed and the effects of the atomic interactions during the expansion of the condensates can be neglected, which holds in the experiment. Since each condensate confined in the

lattice potential is fully coherent, in the frame of single particle theory it can be described by a single order parameter $\Psi_k = (N/(2k_M + 1))^{1/2} \exp[i\theta_k]\phi_k$ according to the Hartree or mean-field approximation [25], where ϕ_k is the single-particle wave function in the kth lattice site and θ_k is the initially random relative phase of the kth condensate. The coefficient $N/(2k_M + 1)$ represents the average particle number of each condensate, with N denoting the total particle number of the whole condensate, and $2k_M + 1$ being the total number of lattice sites.

Following the experimental manipulation sequence in [24], we now consider an atom with two spin states $|0\rangle$ and $|1\rangle$ forming its two logical basis-vectors. Initially, the atom lies in spin state $|0\rangle_k$. Without loss of generality, by using an initial arbitrary α microwave pulse to drive Rabi oscillations between the two spin states, the atom can be placed into a coherent superposition of the two spin states $|0\rangle_k$ and $|1\rangle_k$,

$$
\begin{cases}
|0\rangle_k \rightarrow \cos\left[\dfrac{\alpha}{2}\right] |0\rangle_k + i\sin\left[\dfrac{\alpha}{2}\right] |1\rangle_k\,, \\
|1\rangle_k \rightarrow i\sin\left[\dfrac{\alpha}{2}\right] |0\rangle_k + \cos\left[\dfrac{\alpha}{2}\right] |1\rangle_k\,.
\end{cases}
\tag{1}
$$

After a spin-dependent transport, the spin state of the atom is given by $\cos[\alpha/2] |0\rangle_k + i\exp[i\beta]\sin[\alpha/2] |1\rangle_{k+r}$, where the spatial wave packet of the atom is split into two components in states $|0\rangle_k$ and $|1\rangle_{k+r}$, respectively, i.e., the atomic wave packet is delocalized over the kth and the $(k + r)$th lattice site. In the above notation, the wave packet in state $|0\rangle$ has retained the original lattice site index. Here, r denotes the separation between two wave packets which have originated from the same kth lattice site. The relative phase β between the two wave packets, being independent of the number of particles, is determined by the accumulated kinetic and potential energy phase in the transport process. With the choice of parameters in the experiment, the phase β is almost constant throughout the cloud of atoms and its absolute value is small. Consequently, the atomic wave function can be described by an entangled single-atom state, i.e., an entangled quantum state between the internal degree of freedom (spin) and the external degree of freedom (spatial wave packet)

$$
\psi_k = \cos\left[\frac{\alpha}{2}\right] |0\rangle_k\, \varphi_k + i\exp[i\beta]\sin\left[\frac{\alpha}{2}\right] |1\rangle_{k+r}\, \varphi_{k+r}. \tag{2}
$$

We assume that the spatial wave packet has a form of Gaussian distribution in coordinate space, i.e., $\varphi_k = A\exp[-(x - kd)^2/2\sigma^2]$, where $d = \lambda/2$ is the period of the optical lattice and λ is the wavelength of the retroreflected laser beams. $A = 1/\sigma^{1/2}\pi^{1/4}$ is a normalization constant, and σ denotes the width of the condensate in each optical well. By applying a final $\pi/2$ microwave pulse whose transform rule is given by equation (1), one has

$$
\phi_k = |0\rangle\, \Xi_{0,k} + |1\rangle\, \Xi_{1,k}, \tag{3}
$$

where $\Xi_{0,k}$ and $\Xi_{1,k}$ are respectively given by

$$\Xi_{0,k} = \frac{A}{\sqrt{2}} \left\{ \cos \left[\frac{\alpha}{2}\right] \exp \left[-\frac{(x-kd)^2}{2\sigma^2}\right] \right.$$
$$\left. - \sin \left[\frac{\alpha}{2}\right] \exp \left[i\beta - \frac{(x-(k+r)d)^2}{2\sigma^2}\right] \right\}, \quad (4)$$

$$\Xi_{1,k} = \frac{iA}{\sqrt{2}} \left\{ \cos \left[\frac{\alpha}{2}\right] \exp \left[-\frac{(x-kd)^2}{2\sigma^2}\right] \right.$$
$$\left. + \sin \left[\frac{\alpha}{2}\right] \exp \left[i\beta - \frac{(x-(k+r)d)^2}{2\sigma^2}\right] \right\}. \quad (5)$$

In equation (3), the indices of spin states $|0\rangle$ and $|1\rangle$ are removed in view of the bosonic identity.

Once the spin-dependent optical lattice is switched off, the evolution of the spatial components $\Xi_{j,k}(x,t)$ ($j = 0,1$) of the atomic wave function can be derived by the propagator method [15,16,25]

$$\Xi_{j,k}(x,t) = \int_{-\infty}^{\infty} K(x,t;y,t=0)\Xi_{j,k}(y,t=0)dy, \quad (6)$$

where $\Xi_{j,k}(y,t=0)$ ($j=0,1$) are the spatial components at the initial time $t=0$ which are given by equations (4) and (5), and $K(x,t;y,t=0)$ is the propagator in free space expressed as [26]

$$K(x,t;y,t=0) = \left[\frac{m}{2\pi i\hbar t}\right]^{\frac{1}{2}} \exp \left[\frac{im}{2\hbar t}(x-y)^2\right]. \quad (7)$$

By combining the formulae (4–7), one can obtain the following analytical results of the spatial components after a straightforward calculation:

$$\Xi_{0,k}(x,t) = \frac{A}{\sqrt{2(1+i\gamma t)}} \left\{ \cos \left[\frac{\alpha}{2}\right] \exp \left[-\frac{(x-kd)^2}{2\sigma^2(1+i\gamma t)}\right] \right.$$
$$\left. - \sin \left[\frac{\alpha}{2}\right] \exp \left[i\beta - \frac{(x-(k+r)d)^2}{2\sigma^2(1+i\gamma t)}\right] \right\}, \quad (8)$$

$$\Xi_{1,k}(x,t) = \frac{iA}{\sqrt{2(1+i\gamma t)}} \left\{ \cos \left[\frac{\alpha}{2}\right] \exp \left[-\frac{(x-kd)^2}{2\sigma^2(1+i\gamma t)}\right] \right.$$
$$\left. + \sin \left[\frac{\alpha}{2}\right] \exp \left[i\beta - \frac{(x-(k+r)d)^2}{2\sigma^2(1+i\gamma t)}\right] \right\}, \quad (9)$$

where the parameter $\gamma = \hbar/m\sigma^2$ denotes the trapping frequency within a single well of the optical lattice.

We now consider the density distribution of the overall condensates after switching off the spin-dependent optical lattice. The wave function of the whole sample at time t can be expressed as

$$\Psi(x,t) = \sum_{k=-k_M}^{k_M} \sqrt{\frac{N}{2k_M+1}} \exp[i\theta_k]\phi_k(x,t), \quad (10)$$

where the time-dependent atomic wave function is given by $\phi_k(x,t) = |0\rangle \Xi_{0,k}(x,t) + |1\rangle \Xi_{1,k}(x,t)$, and θ_k denotes the random phase of the kth condensate at time t. In

equation (10), we have neglected the phase diffusion of each condensate possibly induced by quantum and thermal fluctuations, which won't affect the essential of our present problem.

For a Bose-condensed gas confined in a trap, the phase fluctuation is characterized by the fluctuations in the chemical potential [27–29]. In the presence of a 1D optical lattice with sufficiently strong intensity, the phase fluctuations for different condensates in individual wells are independent from each other. Two dominating physical ingredients are responsible for the creation of phase diffusion: one is the collision between condensed atoms and background hot cloud (thermal fluctuation), and the second is spontaneously collective excitation due to quantum fluctuation [29]. In real experiments, the phase diffusion effect is small and can be omitted safely. If the phase diffusion effect was taken into account, the holistic character of the interference pattern will not change except that the central peak of the interference pattern will decrease a little relatively [29].

Obviously, there is no interference between the wave packets in different spin states as the two logical basis-vectors $|0\rangle$ and $|1\rangle$ are orthogonal. The following model is based on an assumption that for the atoms in the entangled single-atom state between the internal and external degrees of freedom each atom interferes only with itself. Concretely, the density distributions of the wave packets in spin states $|0\rangle$ or $|1\rangle$, i.e., the density distributions of the overall condensates confined in the total occupied lattice sites, are not expressed by equations (11) and (12)

$$n_0(x,t) = \left| \sum_{k=-k_M}^{k_M} \sqrt{\frac{N}{2k_M+1}} \exp[i\theta_k]\Xi_{0,k}(x,t) \right|^2, \quad (11)$$

$$n_1(x,t) = \left| \sum_{k=-k_M}^{k_M} \sqrt{\frac{N}{2k_M+1}} \exp[i\theta_k]\Xi_{1,k}(x,t) \right|^2, \quad (12)$$

but given by equations (13) and (14)

$$n_0(x,t) = \frac{N}{2k_M+1} \sum_{k=-k_M}^{k_M} |\Xi_{0,k}(x,t)|^2, \quad (13)$$

$$n_1(x,t) = \frac{N}{2k_M+1} \sum_{k=-k_M}^{k_M} |\Xi_{1,k}(x,t)|^2. \quad (14)$$

The test criterion of this model depends on whether its theoretical prediction accords with the experimental results, i.e., whether this model can interpret well the experiment. Thus we perform a Monte-Carlo analysis of $n_1(x,t)$ by assigning sets of random numbers to the phase $\{\theta_k\}$. Our simulation results show that the interference patterns based on equation (24) don't agree with those observed in the experiment [24] at all. In addition, provide that there are locked phases for individual condensates, i.e., the phase θ_k is not random (a simplest case is that the phase of each condensate is the same), the calculation also shows that the interference patterns derived from equation (12) are not in agreement with the experimental results. Thus

it is implied that equations (11) and (12) are invalid in explaining the experiment. As expected, however, we find that the theoretical interference patterns based on equation (14) agree well with the observed interference patterns in the experiment (see Figs. 1 and 2 in Sect. 3), which indicates that this model, i.e. equations (13) and (14), can be employed to describe the real physics of the emerging interference patterns in the experiment [24].

The physical essence of the density distributions expressed by equations (13) and (14) is that due to the entanglement of a single atom each atom interferes only with itself (or each condensate interferes only with itself), i.e., there is no interference effect between two arbitrary different condensates. In other words, the entanglement of a single atom is sufficient for the interference of the overall condensates and this interference will be irrelevant with the phases of individual condensates.

3 Density distributions and evolution

By using the experimental parameters in [24], we plot the density distributions of the atomic wave packets in states $|0\rangle$ and $|1\rangle$ respectively based on equations (13) and (14).

3.1 Parameters

In the following calculations, the relevant parameters are consistent with those in the experiment, where $\alpha = \pi/2$, $N = 3 \times 10^5$, $\lambda = 785$ nm, and $d = 392.5$ nm. For simplicity, we treat the relative phase β as zero. Nevertheless, we also take into account the effect of it on the interference patterns to compare with the omitted case. Since the value of σ, which characterizes the width of condensate in each lattice site, is chiefly determined by the optical confinement [11], one can evaluate it in terms of a variational calculation. As a result, the ratio $\sigma/d = 0.173$ is obtained. The total number of lattice sites $2k_M + 1$ can be determined theoretically by the formula $k_M^2 = 2\hbar\varpi(15Nad/8\pi^{1/2}a_{ho}\sigma)^{2/5}/(m\omega_x^2 d^2)$ (see Eq. (10) in [11]), where the geometric average of the magnetic frequencies $\varpi_x = 2\pi \times 16$ Hz, m is the mass of ^{87}RGB atom, the oscillator atom, the oscillator length $a_{ho} = \sqrt{\hbar/m\varpi}$, and the s-wave scattering length for ^{87}Rb atom is $a \sim 50$ Å. Thus $k_M \sim 50$ is obtained from the above equation.

3.2 Density distributions in state $|1\rangle$

In order to compare with the experiment, we consider firstly the density distribution of the wave packets in state $|1\rangle$ after the optical lattice is switched off with a time of flight being 14 ms. The analytical result of $n_1(x, t)$ at time t is given by equation (14). Shown in Figure 1a is the density distribution (in units of $H = NA^2/(2k_M+1)$) in state $|1\rangle$ at $t = 14$ ms after initially localized atoms have been delocalized over two lattice sites. Note that in all the figures plotted in this paper the horizontal coordinate x is
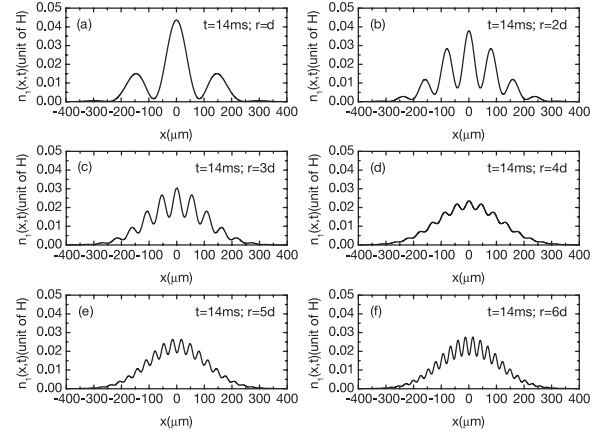

Fig. 1. Density distributions in state $|1\rangle$ after switching off the spin-dependent optical lattice in the cases that initially localized atoms have been delocalized over two (a), three (b), four (c), five (d), six (e), and seven (f) lattice sites by the interferometer sequence (see Fig. 3 in [24]). The time of flight period is 14 ms. The vertical coordinate $n_1(x, t)$ is in units of H ($H = NA^2/(2k_M + 1)$) and the horizontal coordinate x is in units of μm. r denotes the separation between the two wave packets originated from the same lattice site.
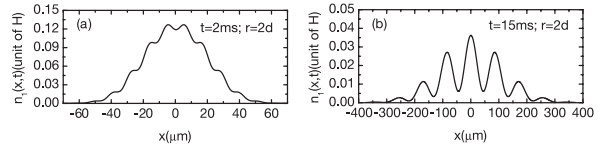

Fig. 2. Evolution of the density distribution in state $|1\rangle$ with time t after switching off the spin-dependent optical lattice in the case that initially localized atoms have been delocalized over three lattice sites. The density distributions are shown at $t = 2$ ms (a) and $t = 15$ ms (b).

in units of μm and the vertical coordinate is in units of $H = NA^2/(2k_M + 1)$. The density distributions in the cases that initially localized atoms have been delocalized over three (b), four (c), five (d), six (e), and seven (f) lattice sites are given in Figures 1b–1f, respectively, where the delocalized extension is denoted by r. With the separation r increasing, we see that the fringe spacing of interference patterns decreases remarkably and the visibility of the interference patterns reduces distinctively (see Fig. 1), which is in agreement with the experimental results (see Fig. 4 in [24]).

In Figure 2, we show the evolution of the density distribution in state $|1\rangle$ after initially localized atoms have been delocalized over three lattice sites. Displayed in Figure 2b is the density distribution at $t = 15$ ms, which agrees well with the observed interference pattern in the experiment (see Fig. 5 in [24]).

In the calculations mentioned above, we have neglected the effect of the relative phase β on the density distribution by treating it as zero. Displayed in Figure 3 are the density distributions for the relative phase β with $-\pi/12$ and $-\pi/3$ respectively. When taking into account the relative phase β between the two wave packets the right-hand
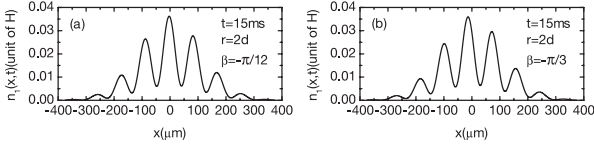
Fig. 3. The effect of the phase β on the density distribution in state $|1\rangle$ after switching off the spin-dependent optical lattice in the case that initially localized atoms have been delocalized over three lattice sites. The time of flight period is 15 ms. The density distributions are shown at $\beta = -\pi/12$ (a) and $\beta = -\pi/3$ (b), respectively.
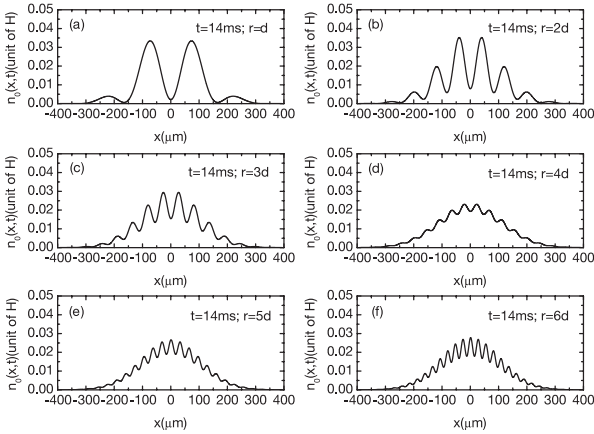

Fig. 4. Density distributions in state $|0\rangle$ after switching off the spin-dependent optical lattice in the cases that initially localized atoms have been delocalized over two (a), three (b), four (c), five (d), six (e), and seven (f) lattice sites. The time of flight period is 14 ms.

side peaks of the density distributions become higher than the left-hand side ones, which breaks the symmetry of the interference patterns to a certain extent. In addition, the larger the absolute value of the phase β is, the weaker the symmetry of the interference pattern becomes. According to the observed interference patterns, we can conclude that the absolute value of the phase β is possibly close to zero in the experiment [24].

3.3 Density distributions in state $|0\rangle$

Now, we discuss the density distributions in state $|0\rangle$ which were not observed in [24]. Similarly to the foregoing analysis, the analytical result of the density distributions in state $|0\rangle$ is given by equation (13). Shown in Figure 4 are the density distributions in state $|0\rangle$ at time $t = 14$ ms. In contrast with the density distributions in state $|1\rangle$, the positions of the sharp peaks in Figures 4a–4f just become those of local minimum densities in Figures 1a–1f and vice versa, which can be interpreted by the conservation of energy and particle number.
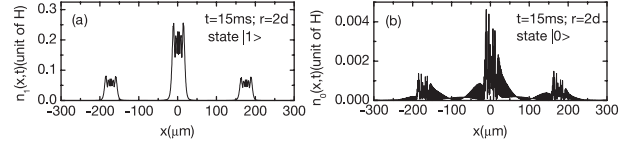

Fig. 5. Density distributions in states $|1\rangle$ (a) and $|0\rangle$ (b) respectively provided that there is a fixed phase distribution among the array of condensates confined in the spin-dependent optical lattice and there exists interference effects between different condensates. The time of flight period is 15 ms. The phase of the kth condensate is given by $\theta_k = 2\pi(k_M + k)/(2k_M + 1)$ ($k = -k_M, ..., k_M$). Here $r = 2$ denotes that initially localized atoms have been delocalized over three lattice sites.

4 Experimental suggestion

As mentioned above, the Bose condensates confined in the 1D spin-dependent optical lattice have no phase coherence relative to each other as the system experienced a Mott transition beforehand. In this situation, the density distributions of the atomic wave packets based on our theoretical model are in good agreement with the observed interference patterns in [24]. From the theoretical model (see Eqs. (13) and (14)), the density distributions of the atomic wave packets in different spin states are irrelevant with the phases of individual condensates in this system. Hence, when there is a locked phase distribution for the array of condensates, the density distributions will not change.

To test further the validity of this model, we propose an experimental suggestion of nonuniform phase distribution. Concretely, we design a fixed linear phase distribution with a total width of 2π for the array of condensates, in which the phase difference between two neighboring condensates is $\delta\theta = 2\pi/(2k_M + 1)$ ($k_M \sim 50$), i.e., the phase of the kth condensate can be expressed by $\theta_k = 2\pi(k_M + k)/(2k_M + 1)$ ($k = -k_M, ..., k_M$). This goal can be achieved by using techniques of phase redistribution such as phase imprinting [30,31] and phase engineering [32,33]. Once a nonuniform phase distribution is performed successfully on the array of condensates, one applies an initial $\alpha = \pi/2$ microwave pulse to drive Rabi oscillations between the two spin states $|0\rangle$ and $|1\rangle$, respectively. Thus all the atoms initially in spin state $|0\rangle$ are placed in a coherent superposition of the two spin states, where the transform rule is given by equation (1).

The following deduction is similar to that in the preceding sections. After a spin-dependent transport and applying a final $\pi/2$ microwave pulse as well as a releasing of the optical lattice, the density distributions of the wave packets in states $|0\rangle$ and $|1\rangle$ respectively would be given by equations (11) and (12) if there were interference effects between different condensates. In this case, the density distributions would be quite different from Figures 1–4. Shown in Figure 5 is the predicted density distribution of the wave packets at time $t = 15$ ms with $r = 2d$ and $\delta\theta = 2\pi/(2k_M + 1)$. From Figure 5, we can see that there exists a strong decay and revival of the density

oscillation, and there is even no visible interference fringe (see Fig. 5b).

Due to the entanglement of a single atom, however, we predict that after the nonuniform phase distribution the density distributions of the wave packets in states $|0\rangle$ and $|1\rangle$ respectively are still given by equations (13) and (14), i.e., the density distributions will not change. In other words, for the atoms in the entangled single-atom state each atom interferes only with itself whether the condensates are fully coherent (with fixed relative phases) or completely independent (with random relative phases), which implies that the entanglement of a single atom is sufficient for the interference of BECs in a spin-dependent optical lattice and the interference effect is irrelevant with the phases of the individual condensates. This experimental proposal provides a straight way to test further our theoretical model and prediction.

5 Discussion and conclusion

To summarize, we have developed a theoretical model to investigate the interference of an array of BECs confined in a 1D spin-dependent optical lattice by calculating the density distributions and evolution of the atomic wave packets. In such a system which has experienced beforehand a Mott transition and a spin-dependent transport, each atom can be described by an entangled single-atom state between the internal (spin) and the external (spatial wave packet) degrees of freedom. Our theoretical model is based on an assumption that for the atoms in the entangled single-atom states each atom interferes only with itself. The results obtained from this model agree well with the interference patterns observed in a recent experiment, which in turn verifies the validity of this model and assumption. In addition, when taking into account the relative phase β between the two wave packets of an atom which is obtained during the transport process, it is found that the symmetry of the density distributions is broken to a certain extent.

From the present work, it has been shown that due to the entanglement of a single atom each atom interferes only with itself (or each condensate interferes only with itself in this system), i.e., there is no interference effect between two arbitrary different condensates. In other words, the entanglement of a single atom is sufficient for the interference of BECs confined in a spin-dependent optical lattice and the interference shows no relevancy with the phases of individual condensates. Finally, an experimental suggestion of nonuniform phase distribution is proposed to test further our theoretical model and prediction. The theoretical model presented here can also be applied to describe the dynamics of BECs trapped in a combined harmonic and optical lattice potential, wherein the number of atoms in individual lattice sites is different. The method could possibly even be extended to consider the case of non-perfect Mott-insulator state.

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